Copy No.

~~CONFIDENTIAL~~

RM No. A7C04

~~46.3~~

~~45~~

copy 2

RESEARCH MEMORANDUM

A PRELIMINARY STUDY OF RAM-ACTUATED COOLING SYSTEMS FOR SUPERSONIC AIRCRAFT

By

Jackson R. Stalder and Kenneth R. Wadleigh

Ames Aeronautical Laboratory
Moffett Field, Calif.

CLASSIFICATION CANCELLED

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

WASHINGTON

April 29, 1947

~~CONFIDENTIAL~~

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

RESEARCH MEMORANDUM

A PRELIMINARY STUDY OF RAM-ACTUATED COOLING SYSTEMS FOR SUPERSONIC AIRCRAFT

By Jackson R. Stalder and Kenneth R. Wadleigh

SUMMARY

An analysis has been made of the characteristics of several cooling cycles suitable for cockpit cooling of supersonic aircraft. All the cycles considered utilize the difference between dynamic and ambient static pressure to actuate the cooling system and require no additional power source.

The results of the study indicate that as flight speeds become greater, increasingly complex systems are required to reduce the ventilating air to tolerable temperatures. At altitudes above approximately 35,000 feet, a system composed of an externally loaded expansion turbine in conjunction with a supersonic diffuser would maintain tolerable ventilating air temperatures, at least up to a flight Mach number of 2. The most complex system considered, composed of a compressor, intercooler, and expansion turbine with the intercooler cooling air decreased in temperature by expansion through an auxiliary turbine is capable of maintaining a ventilating air temperature less than ambient temperature up to a flight Mach number of 3.7. The preceding

[redacted]

results for both systems are predicated on a cockpit pressure equal to ambient static pressure.

It is possible that similar systems can be devised which will allow operation of ram-actuated cooling cycles with the cockpit pressurized, with, however, added system components required in the form of additional heat exchangers and turbines.

INTRODUCTION

It is generally realized that the problem of maintaining habitable cockpit temperatures in airplanes designed for supersonic flight will be difficult. The necessity of cooling the pilot's compartment has arisen in the operation of high-speed subsonic airplanes. The problem will naturally become much more acute as aircraft speeds are increased through the transonic and into the supersonic speed range.

The cooling problem in supersonic aircraft arises, in part, from the near-stagnation temperatures attained in the acceleration of ambient air to velocities approaching that of the airplane which prevail in the boundary layer. In effect, the airplane is surrounded by a thin layer of air at temperatures approaching stagnation value. Solar radiation into the cockpit through the canopy and the dissipation of heat by the pilot and by electrical apparatus further adds to the cooling load. In addition, the entering ventilating air which is at stagnation temperature must be reduced in temperature before admittance to the cockpit.

A refrigeration cycle utilizing atmospheric air as the working medium is currently being employed as a means of enclosure cooling. This cycle uses air that has previously been compressed by the main engine compressor or by a cabin supercharger. The high-pressure high-temperature air is then cooled in an intercooler and expanded through a turbine to the enclosure pressure.

At supersonic flight speeds, the pressure rise occurring from the adiabatic acceleration of the ventilating air becomes of large enough magnitude that it may be possible to utilize the energy of the ram-compressed air to operate a refrigeration cycle as well as for pressurizing the enclosure.

It is the purpose of this report to examine the characteristics of several cooling cycles, which are actuated by the difference between dynamic or ram pressure and ambient static pressure, and to present the results of the analysis in as general a manner as possible. No predictions have been made concerning the magnitude of the enclosure cooling loads as this factor is dependent upon enclosure size, constructional details, amount of insulation employed, etc. Likewise, details of turbine and compressor speeds, sizes, types, have not been discussed in this preliminary report.

SYMBOLS

The following symbols are used throughout the report:

c_p specific heat of air at constant pressure assumed constant at 0.24 Btu per pound, °F

e intercooler cooling effectiveness, dimensionless

hp horsepower

J mechanical equivalent of heat (778 ft-lb/Btu)

M Mach number, dimensionless

P stagnation pressure, pounds per square foot

Q heat abstracted from air by intercooler, Btu per second

T absolute stagnation temperature (°F + 459.7)

W weight flow rate of ventilating air, pounds per second

W' weight flow rate of intercooler cooling air, pounds per second

ϵ adiabatic shaft efficiency, dimensionless

η duct efficiency, dimensionless

γ ratio of specific heats of air (assumed constant at 1.40)

Subscripts

a,b arbitrary stations immediately upstream and downstream, respectively, from component under consideration

c compressor

i ideal or theoretical

o free stream

t turbine

1
1'
2 stations as indicated in figure 1
2'
3
4

ANALYSIS

Five systems have been analyzed. Of these five, four are simple variations of the first basic system (System I) which comprises a supersonic diffuser and an expansion turbine. In this basic system, the air is decelerated to zero velocity (relative to the airplane) in the diffuser and expanded from the resultant high pressure through a turbine to the pressure of the portion of the airplane being cooled – the enclosure. The turbine work is absorbed by an external load such as an electrical generator, hydraulic pump, or similar piece of equipment.

System II is identical with System I except for the addition of a heat exchanger between the diffuser and turbine. The heat exchanger employs an internal cooling medium, that is, fuel, liquid oxygen, or solid CO_2 as the coolant. The turbine work is absorbed by an external load as in System I.

System III employs the turbine work to drive a compressor. The compressor is located downstream from the diffuser and increases the ventilating air pressure above the value of the final diffuser air pressure. An air-to-air intercooler cools the air after compression and before it is expanded through the turbine to the enclosure pressure.

System IV is identical with System III except that the intercooler uses an internal cooling medium rather than free-stream air as a coolant.

System V is similar to System III with, however, the addition of a second turbine which reduces the temperature of the intercooler cooling air. This secondary turbine is directly connected with the main turbine-compressor system so that its work is delivered to the compressor. A schematic diagram of the five systems is shown in figure 1.

For all systems analyzed, the simplifying assumption is made that the pressure drop of the air in passing through a heat exchanger is negligible in comparison with the pressure rise in the diffuser (and compressor when employed) and the pressure drop of the air in passing through the expansion turbine. The accuracy of this assumption is dependent upon the size and geometry of the heat exchanger and the validity of the assumption increases as the flight Mach number increases. Constant values of specific heats of air are also assumed. The air is assumed to be dry so that condensation effects are absent.

The general relations employed in the analysis of the systems are as follows:

The ideal isentropic pressure ratio resulting from deceleration of air from an initial Mach number M_a to zero velocity is given by

$$\left(\frac{P_b}{P_a}\right)_i = \left(\frac{\gamma-1}{2} M_a^2 + 1\right)^{\frac{\gamma}{\gamma-1}} \tag{1}$$

The adiabatic (not necessarily isentropic) temperature ratio under the same conditions is

$$\frac{T_b}{T_a} = \frac{\gamma-1}{2} M_a^2 + 1 \tag{2}$$

Equations (1) and (2) may be derived (reference 1) from consideration of the perfect gas law, the general energy equation, and the equation for sonic velocity in a perfect gas.

A duct efficiency η is defined as the ratio of the actual stagnation pressure rise obtained in the duct to the ideal isentropic stagnation pressure rise obtained for the same entrance Mach number. Thus,

$$\eta = \frac{(P_b - P_a)}{(P_b - P_a)_i} \tag{3}$$

Rearranging equation (3) and substituting equation (1) there is obtained

$$\frac{P_b}{P_a} = 1 + \eta \left[\left(\frac{\gamma-1}{2} M_a^2 + 1\right)^{\frac{\gamma}{\gamma-1}} - 1\right] \tag{4}$$

The intercooler cooling effectiveness e is defined, in the usual manner, as the ratio of the temperature drop of the ventilating air in passing through the cooler to the initial temperature difference between the ventilating air and the cooling air.

The temperature ratio across a compressor or turbine is given, in terms of the pressure ratio, by equations (5) and (6), respectively,

$$\frac{T_b}{T_a} = 1 + \left[\frac{(P_b/P_a)^{\frac{\gamma-1}{\gamma}} - 1}{\epsilon_c}\right] \tag{5}$$

$$\frac{T_b}{T_a} = 1 - \epsilon_t \left[1 - \left(\frac{P_b}{P_a}\right)^{\frac{\gamma-1}{\gamma}}\right] \tag{6}$$

In equation (5), ϵ_c is the adiabatic shaft efficiency defined as the ratio of the isentropic temperature rise to the actual temperature rise of the air for the compressor pressure ratio. In equation (6), ϵ_t is similarly defined as the ratio of the actual drop in temperature experienced by the air as it drops in pressure passing through the turbine to the ideal isentropic temperature drop it would experience for the same turbine pressure ratio.

The horsepower required to drive a compressor is given by

$$hp_c = \frac{Jc_p\ W\ T_a}{550} \left[\frac{(P_b/P_a)^{\frac{\gamma-1}{\gamma}} - 1}{\epsilon_c}\right] \tag{7}$$

and the horsepower delivered by a turbine is

$$hp_t = \frac{J\ c_p\ W\ T_a\ \epsilon_t}{550} \left[1 - \left(\frac{P_b}{P_a}\right)^{\frac{\gamma-1}{\gamma}}\right] \tag{8}$$

The heat abstracted from the air by an internally cooled heat exchanger is

$$Q = W c_p (T_a - T_b) \tag{9}$$

which may be rearranged, in terms of the temperature ratio of the air in passing through the cooler to

$$\frac{T_b}{T_a} = 1 - \frac{Q}{W c_p T_a} \tag{10}$$

The general scheme of analysis is to combine, suitably for each system, the preceding general relationships in order to obtain the ratio of enclosure temperature to ambient static temperature in terms of the flight Mach number, the ratio of enclosure static pressure to free-stream static pressure and the efficiency of the system components.

A detailed analysis of each of the five systems is presented in Appendix A.

DISCUSSION

For all systems, the final temperature ratio and the amount of enclosure pressurization that can be obtained is dependent upon the efficiency with which the diffuser converts the free-stream kinetic energy to static pressure. Available quantitive information on the performance of supersonic diffusers is meager. In reference 2, Kantrowitz and Donaldson have presented a method for the design of reversed DeLaval nozzle-type diffusers and, in addition, have obtained test data over a limited range of Mach numbers to check their analysis. The data of reference 2 was used in this report because of lack of comparable data on other types of diffusers.

It is shown, in reference 2, that it is impossible to obtain a shock-free deceleration through the sonic velocity in a diffuser of this type and that an unavoidable loss in total pressure through the medium of a normal compression shock must result for the flow to be stable. This loss increases with the entrance Mach number. The data of Kantrowitz and Donaldson were used in this report for calculating diffuser efficiencies. The procedure used in determining the diffuser efficiencies used herein was as follows: The maximum diffuser efficiency for a given entrance or flight Mach number was calculated by the method of reference 2. This theoretical maximum efficiency was then multiplied by a factor, 0.93, to obtain an efficiency closely corresponding to the best test efficiencies obtained by Kantrowitz and Donaldson. It is worthy of note that, for a fixed geometry diffuser, the maximum efficiency occurs only at the design Mach number. In this report, it is assumed that the optimum diffuser is used for each flight Mach number. Figure 2 shows the diffuser efficiencies used in this report as calculated by the preceding method.

In order to visualize the magnitude of the stagnation temperatures which occur as a result of the acceleration of ventilating air, figure 3 has been prepared. Figure 3 is derived from equation (2), and shows the stagnation temperature as a function of altitude and Mach number assuming NACA standard air properties. (See reference 3.)

The maximum degree of enclosure pressurization obtainable from flight ram, shown in figure 4, was calculated by means of equation (4) using diffuser efficiencies taken from figure 2. Constant pressure lines for several effective enclosure altitudes are also shown in figure 4. The lower limit of the curves was taken at 30,000 feet altitude, since it is unlikely that prolonged supersonic flight would be undertaken below this altitude. The cooling systems discussed herein would operate less effectively as the enclosure pressure increased and would not operate if the enclosure were maintained at the maximum possible ram pressure. It is probable, however, that systems of this type could be devised which would allow almost complete ram pressurization to be utilized and still maintain the system effectiveness, with, however, the addition of more pieces of equipment - turbines, heat exchangers, etc.

It is unfortunate that the equations for the temperature ratio across each system do not lend themselves to plotting in terms of nondimensional or dimensional groups of variables, so that the effect of a change in efficiency of a system component is immediately apparent. In order to calculate the performance of the system it is necessary to assume values for each of the component efficiencies. The following numerical values were used for substitution in the equation for the temperature ratio across each system:

1. Diffuser efficiency, taken from figure 2
2. Turbine adiabatic efficiency, 0.8
3. Compressor adiabatic efficiency, 0.7
4. Air-to-air intercooler cooling effectiveness, 0.9

The enclosure was assumed to be unpressurized, hence the enclosure pressure was taken as equal to ambient static pressure.

It is thought that the above values approximate the maximum efficiencies that are practically obtainable, considering the probable small size of the equipment. The seemingly high value of intercooler effectiveness arises from the fact that the high ram pressures allow the use of multipass (and hence high effectiveness) heat exchangers.

The performance of System I is shown in figure 5. The pertinent point concerning this sytem is that the final temperature ratio is always greater than unity, that is, the entering ventilating air temperature is always higher than free-stream static temperature, owing to energy losses in the diffuser and turbine.

The temperature ratio of the air after passage through System II is shown in figure 6. In figure 6, the parameter Q/Wc_pT_0 represents the fraction of the initial total heat content of the ambient air that is removed by the internal cooler. This variable is a function of the effectiveness of the internally cooled heat exchanger as well as the lowest temperature obtainable from the cooling medium. From figure 6, it is apparent that if a suitable cooling medium can be employed, it is possible to obtain very low outlet temperatures from this system.

The determination of the characteristics of System III involves, as noted in the appendix, the graphical solution of equations (A9) and (A10) in order to determine the compressor pressure ratio values for subsequent substitution into equation (A8). The values of the compressor pressure ratio determined in the foregoing manner are shown in figure 7. The performance of the system as represented by the final temperature ratio of the ventilating air is presented in figure 8. It is noted that this system is capable of maintaining a system outlet temperature less than free-stream static temperature up to a flight Mach number of approximately 1.7.

The performance of System IV is dependent upon the amount of heat abstracted from the air by the internal cooler, as in System II. The compressor pressure ratios obtained from a graphical solution of equations (A13) and (A14) are shown in figure 9. The pressure ratios decrease rapidly as heat is abstracted by the internal cooler due to the decreased turbine work available with the lower turbine inlet temperatures. The final over-all temperature ratio across the system is shown in figure 10. It may be seen from this figure that the final temperature ratio is determined by the amount of heat abstracted by the internal cooler and that the system is sensitive to changes in flight Mach number since a slight change in Mach number increases the final temperature ratio considerably.

The determination of the performance of System V is complicated by the presence of the auxiliary cooling turbine that decreases the temperature of the cooling air before its passage through the intercooler. It becomes necessary to assume values for the mass flow ratio of intercooler cooling air to ventilating air, as may be seen from an examination of equation (A18). In order to obtain the system temperature ratio as represented by equation (A17) it was necessary to solve, graphically, equations (A18) and (A19) for values of the compressor pressure ratio. The numerical values obtained are shown in figure 11. From a consideration of figures 11 and 12, it is evident that the compressor pressure ratios become extremely high at Mach numbers greater than 2.5 for a value of W'/W of 2. It will be noted that these pressure ratios are considerably higher than those obtained in Systems III and IV, due to the added work put into the compressor by the auxiliary turbine. The auxiliary turbine performs two beneficial functions – it increases the work available to drive the compressor and, hence, increases the pressure ratio across the main turbine, and it provides cold exhaust air to cool the primary ventilating air.

The final ventilating-air-temperature ratio across System V is shown in figure 12. The performance of this system shows a marked decrease in over-all temperature ratio compared to that of the other systems due to the beneficial effect of the auxiliary turbine. From figure 12 it is seen that, for a value of mass-flow ratio of cooling to ventilating air of 1.5, the system outlet temperature may be maintained less than, or equal to, ambient static temperature up to a Mach number of 3.7. The added performance of this system is, of course, accompanied by an increase in the power required to ram the additional cooling air through the system.

The fact that the temperature ratio decreases to a minimum at a Mach number of approximately 1.9 and then increases as the flight Mach number increases is due to the opposing effects of increasing pressure ratio across the auxiliary turbine and decreasing duct efficiency.

For all the systems discussed, it is strongly emphasized that the performance shown is the maximum obtainable with the assumed values of compressor and turbine efficiencies and intercooler effectiveness, because the maximum obtainable value of diffuser efficiency for the type of diffuser considered was taken for each flight Mach number. The performance over a range of flight Mach numbers of any actual enclosure cooling system with a fixed geometry diffuser of the type discussed would decrease below the ideal values shown herein. The values of diffuser efficiency used herein should not be considered the maximum obtainable with any type of diffuser. It is quite possible that diffusers can be designed which will have performances superior to the simple reversed DeLaval-nozzle type discussed.

Systems, such as those described in this report, which take free stream air and pass it through a cooling cycle contribute drag to the airplane by virtue of the difference in momentum between the air taken into, and that discharged from, the airplane. Thus, the choice of a particular system will depend upon the system drag characteristics as well as upon the internal efficiency of the installation. The drag produced by a system is greatly dependent upon the size and configuration of the ducts and heat exchangers, hence it is not possible to draw specific conclusions concerning the drag of the several systems discussed herein. However, it is apparent that cooling systems that handle relatively large amounts of cooling air such as Systems III and V will have unfavorable drag characteristics and it may be necessary to reduce the heat-exchanger effectiveness in order to eliminate excessive pressure drops on the cooling air side. Detailed computations required to design an optimum system would require, as a starting point, specifications concerning flight speeds, altitudes, and cooling load, and hence have not been undertaken in this general report.

CONCLUDING REMARKS

The high ventilating and boundary-layer-air temperatures attained at supersonic velocities will probably require use of an enclosure cooling system to permit piloted supersonic aircraft to be flown.

[redacted]

An examination of the characteristics of several ram-actuated cooling systems discloses that these systems may, from a standpoint of cooling performance, be suitable for the cooling of aircraft or missiles operating at supersonic velocities. The operation of all the systems depends upon the difference in pressure between the enclosure or cockpit and the ram pressures due to the velocity of the aircraft.

A system (System V) composed of a supersonic diffuser, a compressor, intercooler, and two expansion turbines appears promising, from a cooling standpoint, in view of the fact that the ideal performance of the system indicates a system outlet temperature less than ambient static temperature up to a flight Mach number of 3.7, provided the enclosure is maintained at ambient static pressure.

Ames Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Moffett Field, Calif.

Jackson R. Stalder
Jackson R. Stalder,
Mechanical Engineer.

Kenneth R. Wadleigh,
Mechanical Engineer.

Approved:

Smith J. DeFrance
Smith J. DeFrance,
Engineer-in-Charge.

[redacted]

[redacted]

APPENDIX

System I

For this basic system, equations (2), (4), and (6) are combined to give the final temperature ratio across the system in terms of the initial (flight) Mach number, diffuser efficiency, turbine efficiency, and the ratio of enclosure static pressure to free-stream static pressure.

The latter ratio is, for a given system and flight Mach number, dependent upon the amount of enclosure pressurization desired. Combination of equations (2), (4), and (6) gives, for the final temperature ratio across the system,

$$\frac{T_2}{T_0} = \left(\frac{T_1}{T_0}\right)\left(\frac{T_2}{T_1}\right) \qquad \text{(A1)}$$

or

$$\frac{T_2}{T_0} = \left(1 + \frac{\gamma-1}{2} M_0^2\right)\left\{1 - \epsilon_t\left[1 - \left(\frac{P_2/P_0}{\left\{1 + \eta\left[\left(\frac{\gamma-1}{2} M_0^2 + 1\right)^{\frac{\gamma}{\gamma-1}} - 1\right]\right\}}\right)^{\frac{\gamma-1}{\gamma}}\right]\right\} \qquad \text{(A2)}$$

[redacted]

[redacted]

[redacted]

The power available from the turbine, which is absorbed by an external load, is,

$$hp_t = \left(\frac{Jc_pWT_1\epsilon_t}{550}\right)\left(1-\left\{\frac{P_2/P_0}{1+\eta\left[\left(\frac{\gamma-1}{2}M_0^2+1\right)^{\frac{\gamma}{\gamma-1}}-1\right]}\right\}^{\frac{\gamma-1}{\gamma}}\right) \qquad (A3)$$

System II

The over-all final temperature ratio across the system is given by,

$$\frac{T_3}{T_0} = \left(\frac{T_1}{T_0}\right)\left(\frac{T_2}{T_1}\right)\left(\frac{T_3}{T_2}\right) \qquad (A4)$$

Substituting values for $\frac{T_1}{T_0}$, $\frac{T_2}{T_1}$, and $\frac{T_3}{T_2}$ there is obtained

$$\frac{T_3}{T_0} = \left(\frac{\gamma-1}{2}M_0^2+1\right)\left[1-\frac{Q}{Wc_pT_0\left(\frac{\gamma-1}{2}M_0^2+1\right)}\right]\left\{1-\epsilon_t\left[1-\left\{\frac{P_3/P_0}{1+\eta\left[\left(\frac{\gamma-1}{2}M_0^2+1\right)^{\frac{\gamma}{\gamma-1}}-1\right]}\right\}^{\frac{\gamma-1}{\gamma}}\right]\right\} \qquad (A5)$$

[redacted]

The turbine power, absorbed by an external load is, from equation (8)

$$hp_t = \frac{Jc_pW\epsilon_t}{550}\left[T_o\left(\frac{\gamma-1}{2} M_o^2 +1\right) - \frac{Q}{Wc_p}\right]\left[1-\left\{\frac{P_3/P_o}{1+\eta\left[\left(\frac{\gamma-1}{2} M_o^2 +1\right)^{\frac{\gamma}{\gamma-1}}-1\right]}\right\}^{\frac{\gamma-1}{\gamma}}\right] \tag{A6}$$

System III

Equations (2), (4), (5), and (6), when substituted into the expression for the final temperature ratio across the system

$$\frac{T_4}{T_o} = \left(\frac{T_1}{T_o}\right)\left(\frac{T_2}{T_1}\right)\left(\frac{T_3}{T_2}\right)\left(\frac{T_4}{T_3}\right) \tag{A7}$$

give

$$\frac{T_4}{T_o} = \left(\frac{\gamma-1}{2} M_o^2 +1\right)\left\{\left[1+\frac{(P_2/P_1)^{\frac{\gamma-1}{\gamma}}-1}{\epsilon_c}\right](1-c)+c\right\}\left\{1-\epsilon_t\left[1-\left\{\frac{P_4/P_o}{1+\eta\left[\left(\frac{\gamma-1}{2} M_o^2+1\right)^{\frac{\gamma}{\gamma-1}}-1\right]}\right\}^{\frac{\gamma-1}{\gamma}}\left(\frac{1}{P_2/P_1}\right)^{\frac{\gamma-1}{\gamma}}\right]\right\} \tag{A8}$$

It will be noted that equation (A8) contains the term P_2/P_1, the compressor pressure ratio. This term may be eliminated by equating the expressions for compressor and turbine powers since all of the turbine power is delivered to the compressor. Hence, when equations (7) and (8) are equated and the resultant expression is solved for the term P_2/P_1 the following quadratic solution results:

$$\frac{P_2}{P_1} = \left[\frac{\left(1+\frac{T_1}{T_3\epsilon_c\epsilon_t}\right) \pm \sqrt{\left(1+\frac{T_1}{T_3\epsilon_c\epsilon_t}\right)^2 - 4\left(\frac{T_1}{T_3\epsilon_c\epsilon_t}\right)\left\{\frac{P_4/P_0}{1+\eta\left[\left(\frac{\gamma-1}{2}M_0^2+1\right)^{\frac{\gamma}{\gamma-1}}-1\right]}\right\}^{\frac{\gamma-1}{\gamma}}}}{2\left(\frac{T_1}{T_3\epsilon_c\epsilon_t}\right)}\right]^{\frac{\gamma}{\gamma-1}} \tag{A9}$$

Values of the term T_1/T_3 in equation (A9), are obtained from the following expression which is derived from equation (5) and the expression for intercooler effectiveness,

$$\frac{T_1}{T_2} = \frac{1}{\left\{ e + (1-e)\left[1 + \frac{(P_2/P_1)^{\frac{\gamma-1}{\gamma}} - 1}{\epsilon_c}\right]\right\}} \tag{A10}$$

To obtain values of the term P_2/P_1 for substitution into equation (A8), a graphical simultaneous solution of equations (A9) and (A10) is made. Values of the term P_4/P_0 may be chosen for any degree of enclosure pressurization desired.

System IV

The final temperature ratio across the system is,

$$\frac{T_4}{T_0} = \left(\frac{T_1}{T_0}\right)\left(\frac{T_2}{T_1}\right)\left(\frac{T_3}{T_2}\right)\left(\frac{T_4}{T_3}\right) \tag{A11}$$

or, in terms of the system variables,

$$\frac{T_4}{T_0}=\left(\frac{\gamma-1}{2}M_0^2+1\right)\left[1+\frac{(P_2/P_1)^{\frac{\gamma-1}{\gamma}}-1}{\epsilon_c}\right]\left\{1-\frac{\left(\frac{Q}{Wc_pT_0}\right)}{\left(1+\frac{\gamma-1}{2}M_0^2\right)\left[1+\frac{(P_2/P_1)^{\frac{\gamma-1}{\gamma}}-1}{\epsilon_c}\right]}\right\}$$

$$\left\{1-\epsilon_t\left[1-\left(\frac{P_4/P_0}{P_2/P_1}\right)^{\frac{\gamma-1}{\gamma}}\left\{\frac{1}{1+\eta\left[\left(\frac{\gamma-1}{2}M_0^2+1\right)^{\frac{\gamma}{\gamma-1}}-1\right]}\right\}^{\frac{\gamma-1}{\gamma}}\right]\right\} \qquad \text{(A12)}$$

The unknown compressor pressure ratio P_2/P_1 , in equation (A12), is determined in the same manner as for System III. The resulting equations involving P_2/P_1 are

$$\frac{P_2}{P_1} = \left[\frac{1+\dfrac{T_1}{T_3\,\epsilon_c\,\epsilon_t} \pm \sqrt{\left(1+\dfrac{T_1}{T_3\,\epsilon_c\,\epsilon_t}\right)^2 - 4\left(\dfrac{T_1}{T_3\,\epsilon_c\,\epsilon_t}\right)\left\{\dfrac{P_4/P_0}{1+\eta\left[\left(\dfrac{\gamma-1}{2}M_0^2+1\right)^{\frac{\gamma}{\gamma-1}}-1\right]}\right\}^{\frac{\gamma-1}{\gamma}}}}{2\left(\dfrac{T_1}{T_3\,\epsilon_c\,\epsilon_t}\right)} \right]^{\frac{\gamma}{\gamma-1}} \tag{A13}$$

and

$$\frac{T_1}{T_3} = \left\{\frac{1}{1+\left[\dfrac{(P_2/P_1)^{\frac{\gamma-1}{\gamma}}-1}{\epsilon_c}\right]}\right\}\left[\frac{1}{1-\left(\dfrac{Q}{Wc_pT_0}\right)\left(\dfrac{1}{1+\dfrac{\gamma-1}{2}M_0^2}\right)\left\{\dfrac{1}{1+\left[\dfrac{(P_2/P_1)^{\frac{\gamma-1}{\gamma}}-1}{\epsilon_c}\right]}\right\}}\right] \tag{A14}$$

[redacted]

Equations (A13) and (A14) are simultaneously solved for values of P_2/P_1 which are subsequently substituted into equation (A12). It will be noted that in this sytem, an additional independent variable Q/Wc_pT_0 is introduced. This variable is determined by the amount of heat abstracted from the air by the intercoolor and may be arbitrarily chosen to yield the desired final temperature ratio.

System V

The analysis of this system is similar to that of System III except that the intercoolor equation must be modified as the cooling air is now precooled by expansion through a turbine. The expression for intercooler effectiveness is, for this system,

$$e = \frac{T_2 - T_3}{T_2 - T_2'} \tag{A15}$$

The assumption is made that the precooling turbine has the same adiabatic efficiency as the main turbine. The expression for the over-all final temperature ratio across the system then is

$$\frac{T_4}{T_0} = \left(\frac{T_1}{T_0}\right)\left(\frac{T_2}{T_1}\right)\left(\frac{T_3}{T_2}\right)\left(\frac{T_4}{T_3}\right) \tag{A16}$$

or

in terms of the system variables,

$$\frac{T_4}{T_0} = \left(\frac{\gamma-1}{2} M_0{}^2 + 1\right)\left[(1-e)\left[1+\frac{(P_2/P_1)^{\frac{\gamma-1}{\gamma}}-1}{\epsilon_c}\right] + e\left\{1-\epsilon_t\left[1-\left(\frac{1}{P_1/P_0}\right)^{\frac{\gamma-1}{\gamma}}\right]\right\}\right]\left\{1-\epsilon_t\left[1-\left(\frac{P_4/P_0}{\frac{P_2}{P_1}\,\frac{P_1}{P_0}}\right)^{\frac{\gamma-1}{\gamma}}\right]\right\} \tag{A17}$$

where the term P_1/P_0 is taken from equation (4). In the derivation of equation (A17) it is assumed that $P_2{}'$ is equal to P_0, that is, the intercooler pressure drop is considered negligible.

The unknown compressor pressure ratio P_2/P_1 in equation (A17), is determined by equating the compressor power to the sum of the powers of the main and precooling turbines. The resulting expression for the term P_2/P_1 is

$$\frac{P_2}{P_1} = \left[\frac{\left[1+\frac{T_1}{T_3}\left\{\frac{W'}{W}\left[1-\left(\frac{1}{P_1/P_0}\right)^{\frac{\gamma-1}{\gamma}}\right]+\frac{1}{\epsilon_t\epsilon_c}\right\}\right] \pm \sqrt{\left[1+\frac{T_1}{T_3}\left\{\frac{W'}{W}\left[1-\left(\frac{1}{P_1/P_0}\right)^{\frac{\gamma-1}{\gamma}}\right]+\frac{1}{\epsilon_t\epsilon_c}\right\}\right]^2 - \frac{4T_1}{T_3\epsilon_t\epsilon_c}\left(\frac{P_4/P_0}{P_1/P_0}\right)^{\frac{\gamma-1}{\gamma}}}}{\frac{2\,T_1}{T_3\,\epsilon_t\,\epsilon_c}}\right]^{\frac{\gamma}{\gamma-1}} \qquad \text{(A18)}$$

Again, a graphical solution is required in order to obtain values for the term P_2/P_1 because the expression for the term T_1/T_3 contains the term P_2/P_1, thus,

$$\frac{T_1}{T_3} = \left(\frac{T_1}{T_2}\right)\left(\frac{T_2}{T_3}\right)$$

or

$$\frac{T_1}{T_3} = \left[\frac{1}{1+\frac{(P_2/P_1)^{\frac{\gamma-1}{\gamma}}-1}{\epsilon_c}}\right]\left[\frac{1}{1-e\left\{1-\frac{1-\epsilon_t\left[1-\left(\frac{1}{P_1/P_0}\right)^{\frac{\gamma-1}{\gamma}}\right]}{1+\left[\frac{(P_2/P_1)^{\frac{\gamma-1}{\gamma}}-1}{\epsilon_c}\right]}\right\}}\right] \qquad \text{(A19)}$$

In the simultaneous graphical solution of equations (28) and (29), it is necessary to assume values of the mass-flow ratio of intercooler cooling air to ventilating air W'/W. In the actual case, the value of this parameter is determined by the area of the intercooler heat-transfer surface and the over-all heat-transfer coefficient obtained in the cooler, since the cooling effectiveness is arbitrarily chosen.

The presentation of the results of the foregoing analysis in the form of a final system temperature ratio is believed to be the most convenient form of presentation because temperature changes due to variation in altitude then do not enter the equations.

The cooling capacity of any system may be calculated for any desired flight Mach number and altitude by the following method:

The ambient temperature is multiplied by the system temperature ratio to obtain the temperature of the ventilating air entering the enclosure. The difference between the enclosure ambient temperature and the entering ventilating air temperature is then the temperature difference available for cooling the enclosure. Multiplication of the temperature difference by the term Wc_p gives the cooling capacity of the system in terms of Btu per second.

REFERENCES

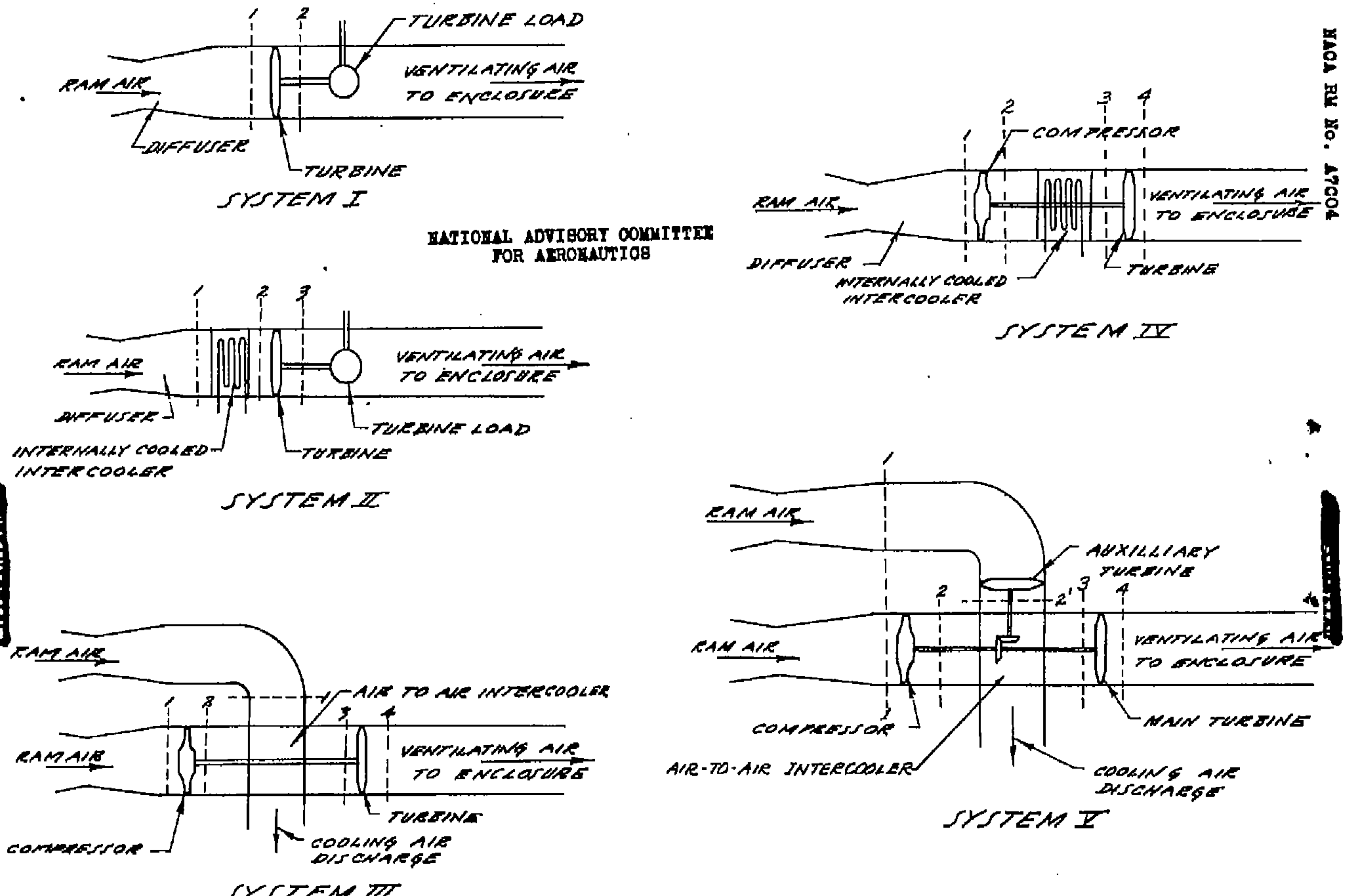
1. Bailey, Neil P.: The Thermodynamics of Air at High Velocities. Jour. Aero. Sci., vol. 11, no. 3, July, 1944.

2. Kantrowitz, Arthur and Donaldson, Coleman duP.: Preliminary Investigation of Supersonic Diffusers. NACA ACR No. L5D20, 1945.

3. Brombacker, W.G.: Altitude-Pressure Tables Based on the United States Standard Atmosphere. NACA Rep. No. 538, 1935.

NATIONAL ADVISORY COMMITTEE
FOR AERONAUTICS

[redacted]



(a) ENCLOSURE COOLING SYSTEMS I, II, AND III.

FIGURE 1.- SCHEMATIC DIAGRAM OF ENCLOSURE COOLING SYSTEMS.

(b) ENCLOSURE COOLING SYSTEMS IV AND V.

FIGURE 1.- CONCLUDED.

[redacted]

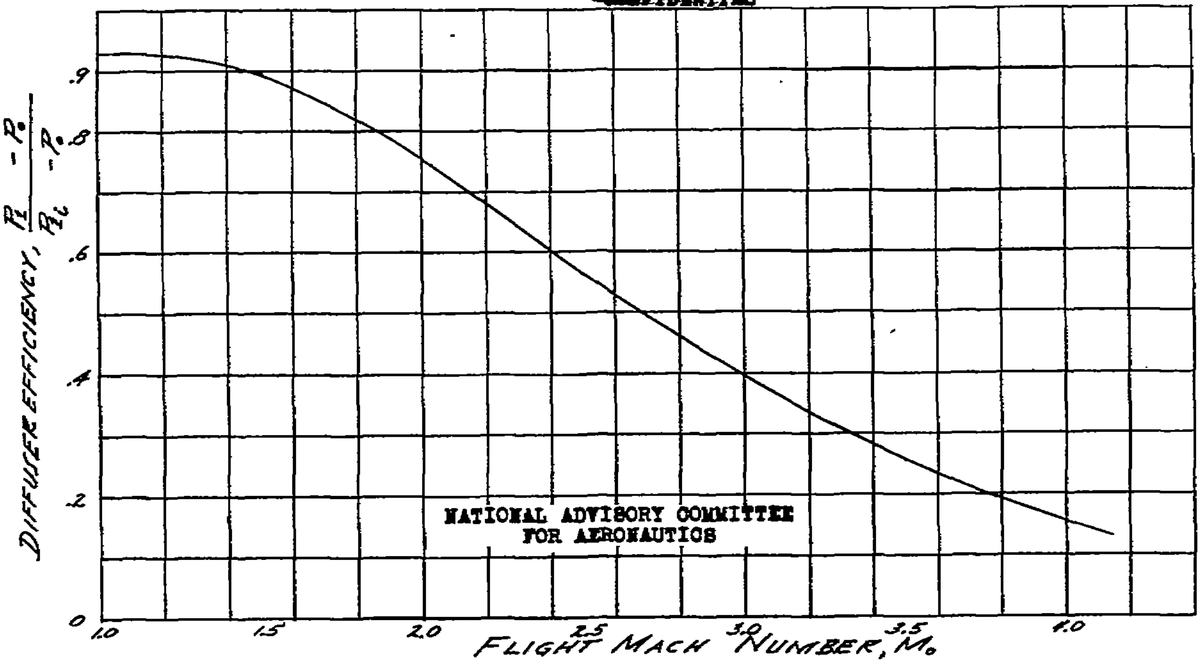


FIGURE 2.- MAXIMUM COMPUTED SUPERSONIC DIFFUSER EFFICIENCY WITH ALLOWANCE FOR PRESSURE LOSS IN SUBSONIC PORTION OF DIFFUSER AS A FUNCTION OF FLIGHT MACH NUMBER. MAXIMUM EFFICIENCIES COMPUTED FROM DATA OF REFERENCE 2.

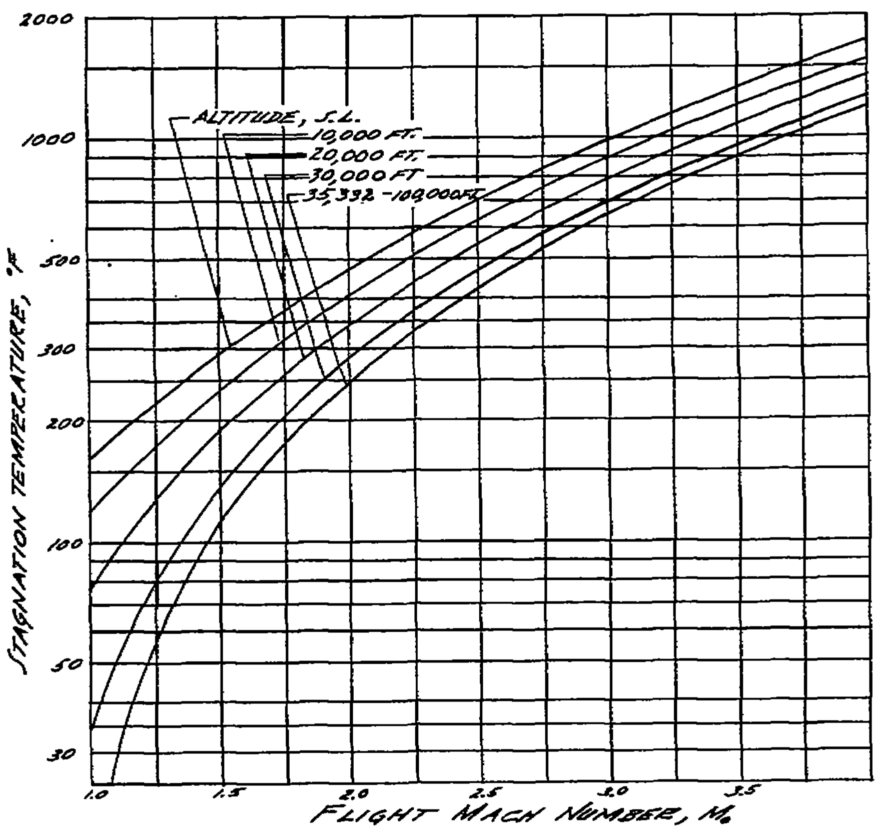


FIGURE 3.- RELATION BETWEEN FLIGHT MACH NUMBER, ALTITUDE, AND STAGNATION TEMPERATURE FOR NACA STANDARD ATMOSPHERE. [redacted]

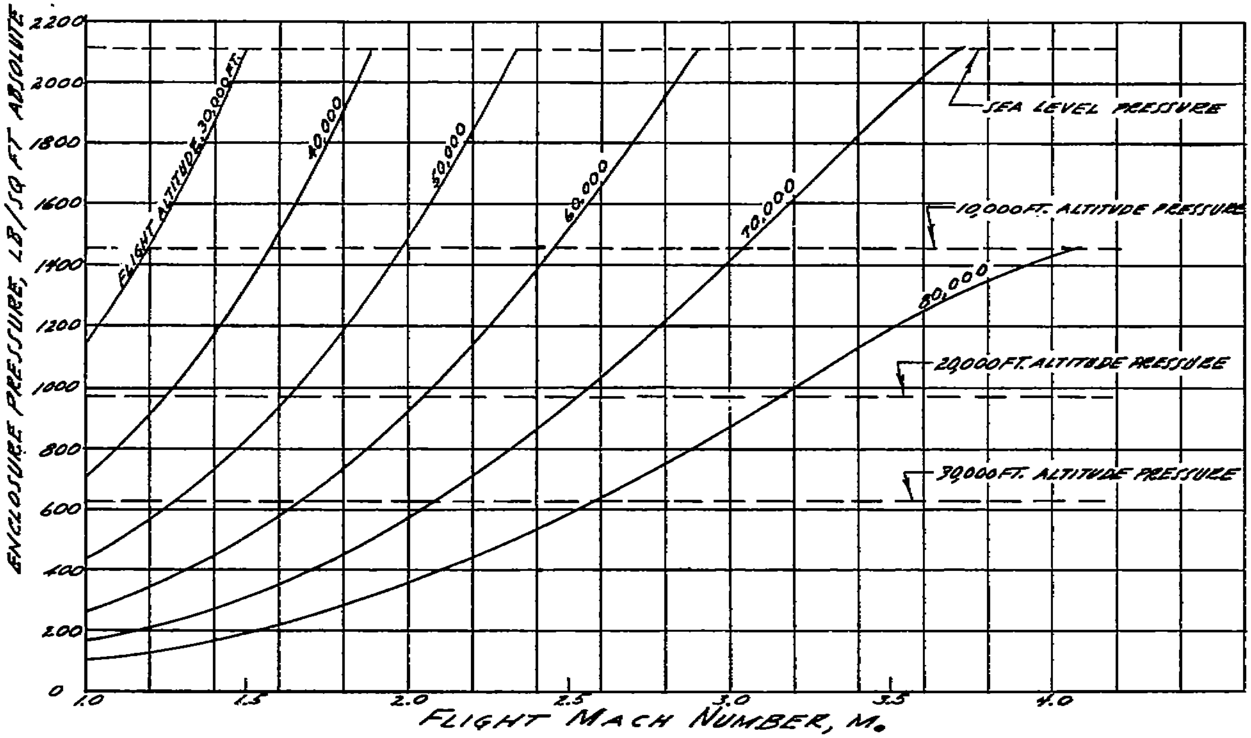


FIGURE 4.- ENCLOSURE PRESSURIZATION OBTAINABLE BY USE OF RAM PRESSURE AS A FUNCTION OF FLIGHT ALTITUDE AND MACH NUMBER. DUCT EFFICIENCY TAKEN FROM FIG. 2.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

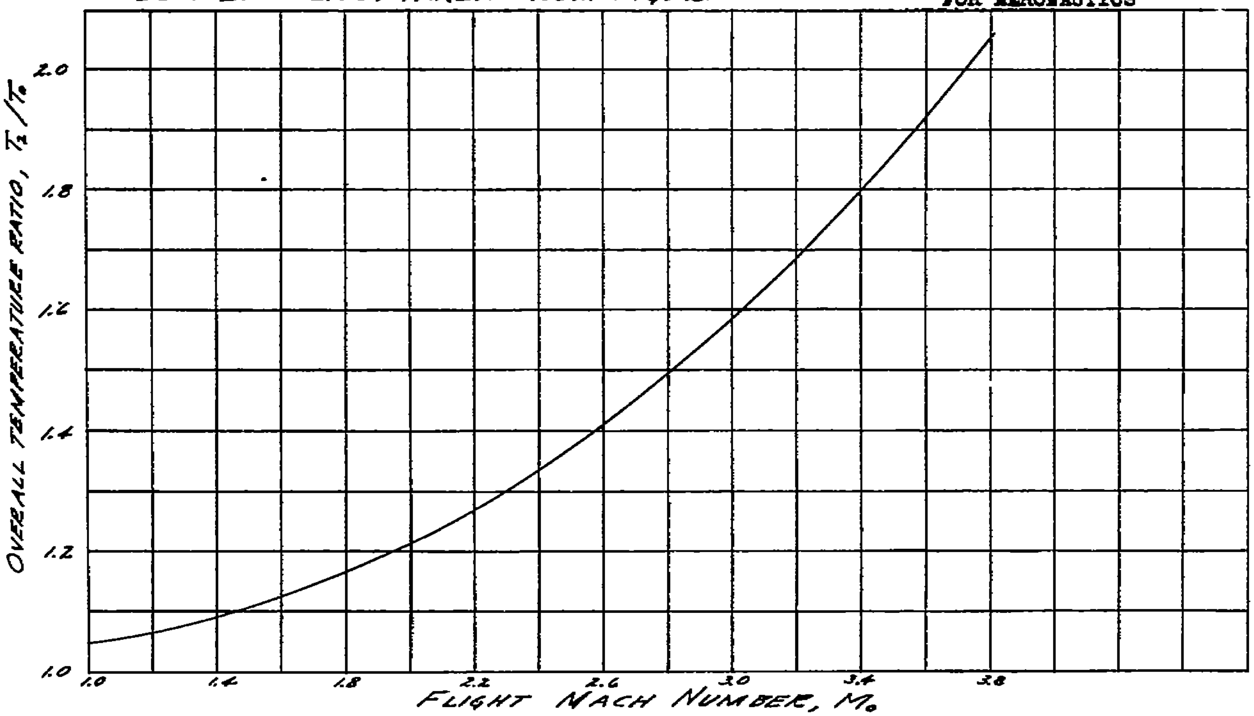


FIGURE 5.- OVERALL TEMPERATURE RATIO OF VENTILATING AIR FOR SYSTEM I AS A FUNCTION OF FLIGHT MACH NUMBER. TURBINE ADIABATIC EFFICIENCY 80%; DUCT EFFICIENCY TAKEN FROM FIG. 2; ENCLOSURE PRESSURE EQUAL TO AMBIENT STATIC PRESSURE.

[redacted]

[redacted]

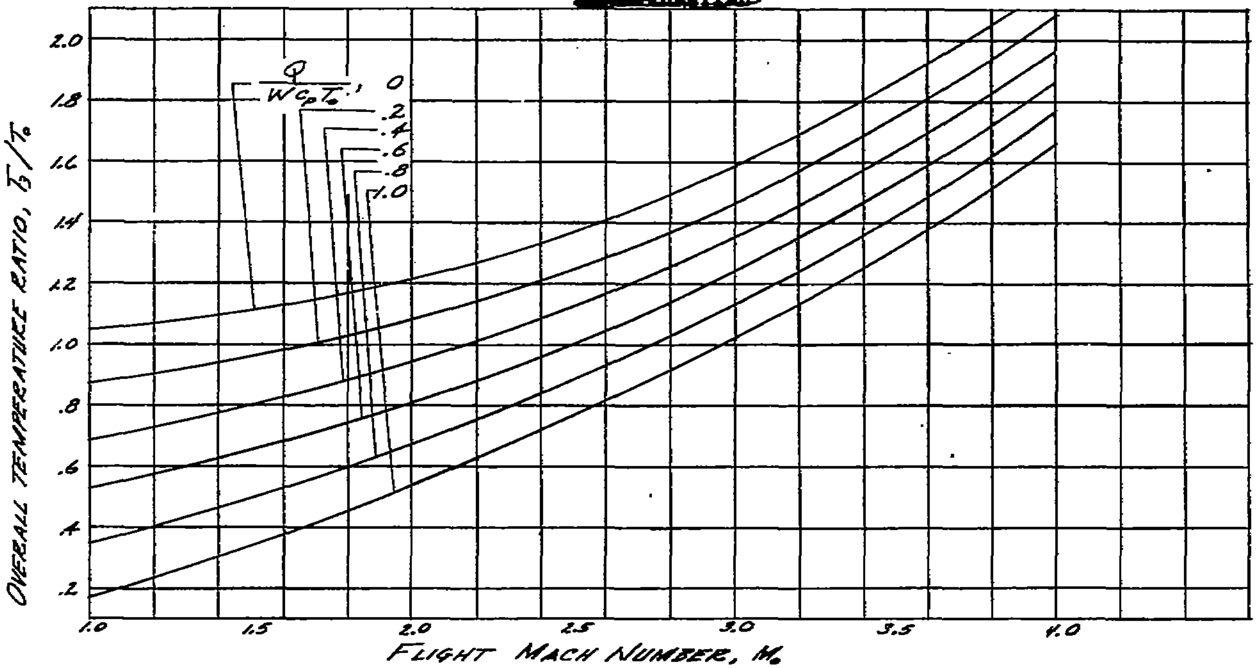


FIGURE 6.- OVERALL TEMPERATURE RATIO OF VENTILATING AIR FOR SYSTEM II AS A FUNCTION OF FLIGHT MACH NUMBER AND RATIO OF HEAT ABSTRACTED BY INTERCOOLER TO FREE STREAM HEAT CONTENT. ENCLOSURE PRESSURE EQUAL TO AMBIENT STATIC PRESSURE; TURBINE ADIABATIC EFFICIENCY, 80%; DUCT EFFICIENCY TAKEN FROM FIG. 2.

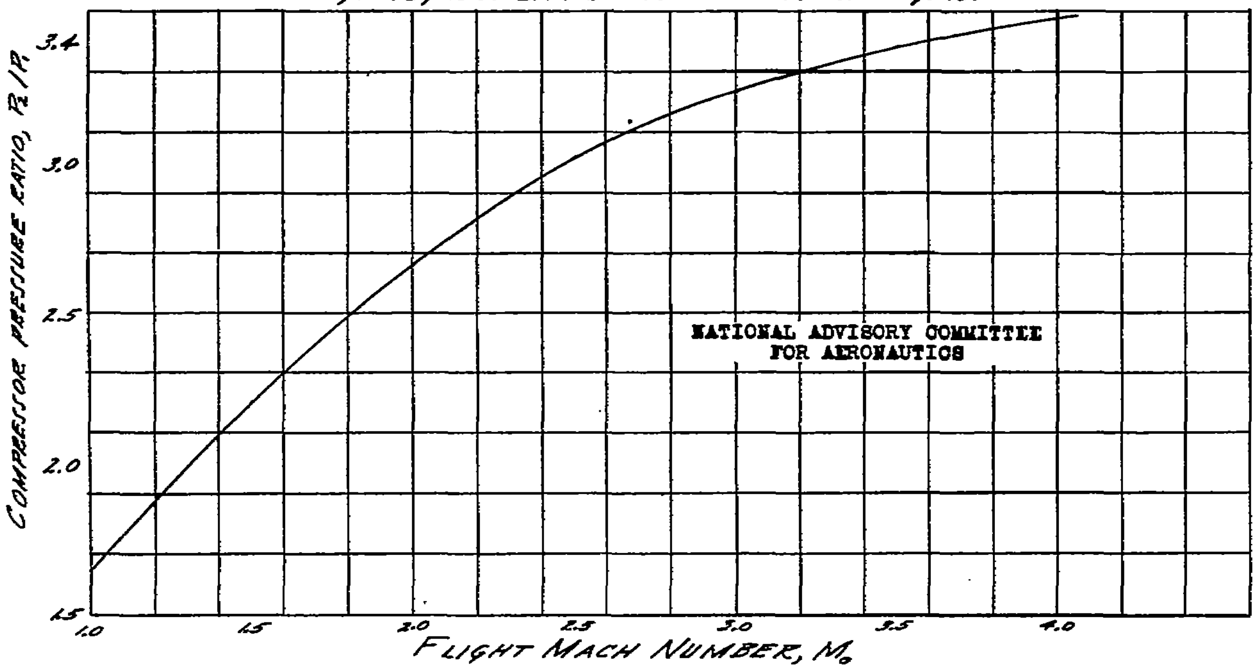


FIGURE 7.- COMPRESSOR PRESSURE RATIO AS A FUNCTION OF FLIGHT MACH NUMBER FOR SYSTEM III. ENCLOSURE PRESSURE EQUAL TO AMBIENT STATIC PRESSURE; COMPRESSOR ADIABATIC EFFICIENCY, 70%; TURBINE ADIABATIC EFFICIENCY, 80%; INTERCOOLER EFFECTIVENESS, 90%; DUCT EFFICIENCY TAKEN FROM FIG. 2.

[redacted]

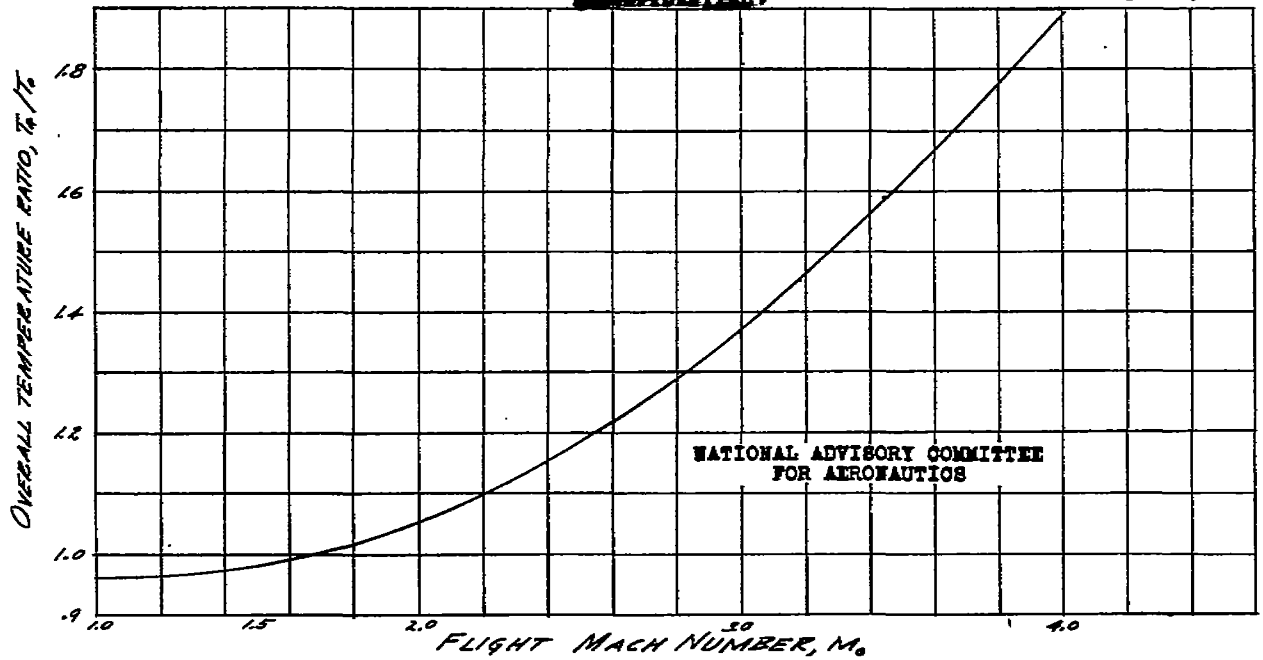


FIGURE 8.- OVERALL TEMPERATURE RATIO OF VENTILATING AIR FOR SYSTEM III AS A FUNCTION OF FLIGHT MACH NUMBER. TURBINE ADIABATIC EFFICIENCY, 80%; COMPRESSOR ADIABATIC EFFICIENCY, 70%; INTERCOOLER EFFECTIVENESS 90%; DUCT EFFICIENCY TAKEN FROM FIGURE 2.

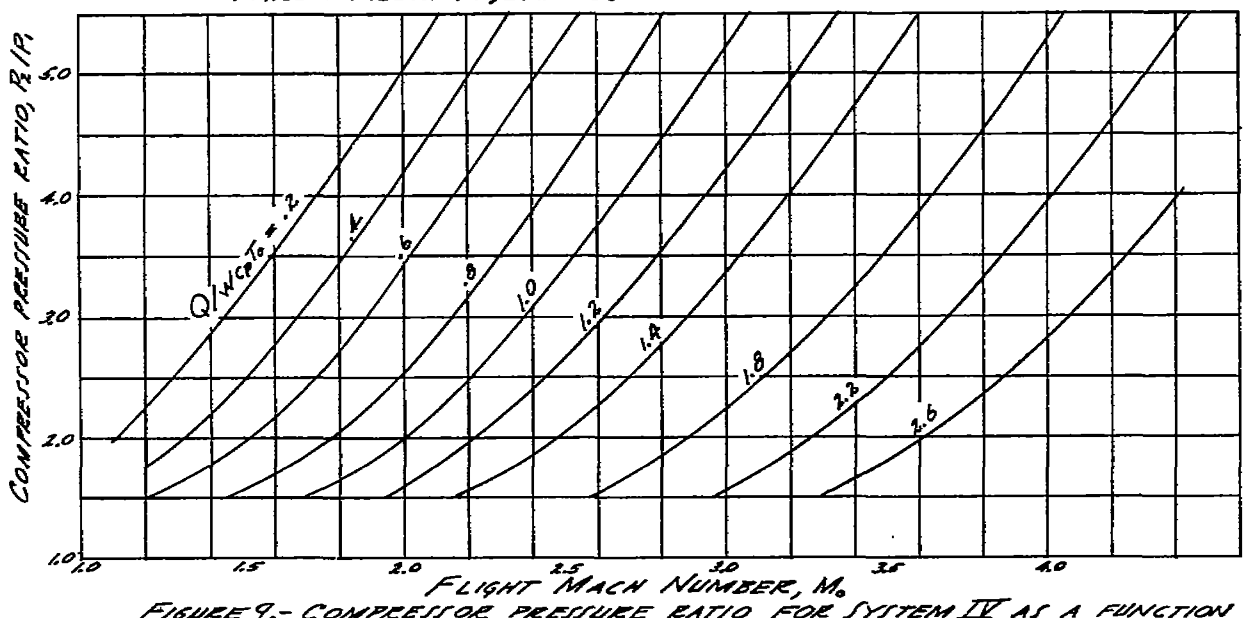


FIGURE 9.- COMPRESSOR PRESSURE RATIO FOR SYSTEM IV AS A FUNCTION OF FLIGHT MACH NUMBER AND RATIO OF HEAT ABSTRACTED BY INTERCOOLER TO FREE STREAM HEAT CONTENT. ENCLOSURE PRESSURE EQUAL TO AMBIENT STATIC PRESSURE; TURBINE ADIABATIC EFFICIENCY, 80%; COMPRESSOR ADIABATIC EFFICIENCY, 70%; DUCT EFFICIENCY TAKEN FROM FIG. 2.

[redacted]

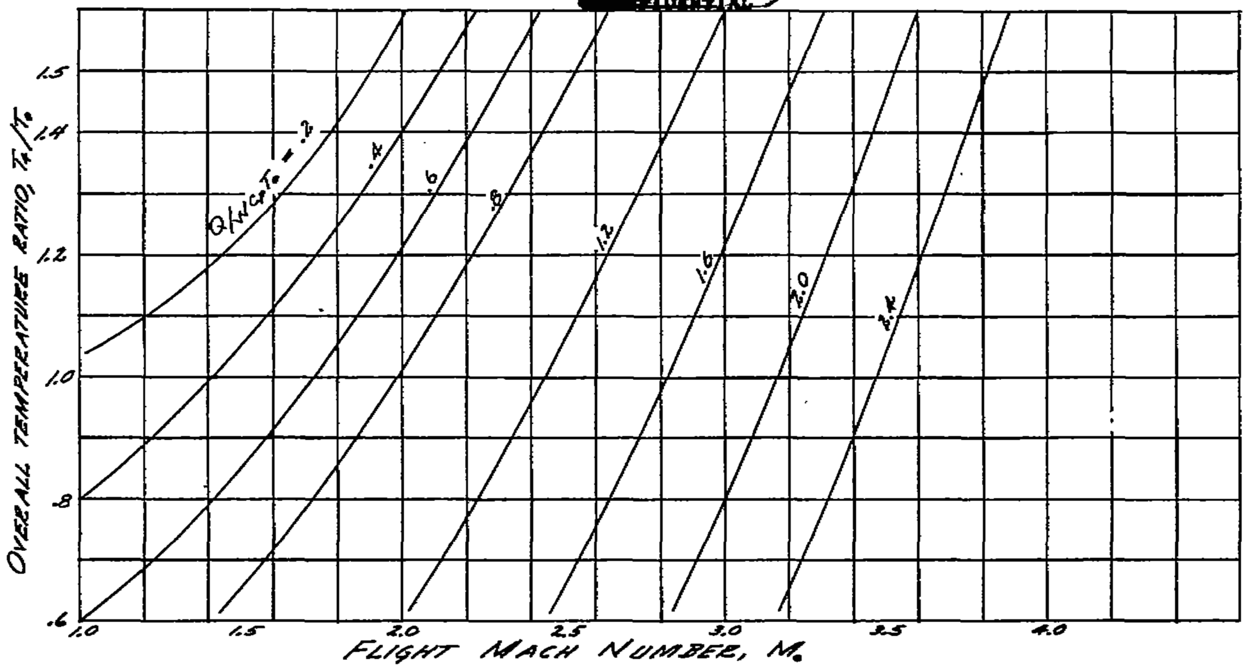


FIGURE 10.- OVERALL TEMPERATURE RATIO OF VENTILATING AIR FOR SYSTEM IV AS A FUNCTION OF FLIGHT MACH NUMBER AND RATIO OF HEAT ABSTRACTED BY INTERCOOLER TO FREE STREAM HEAT CONTENT. ENCLOSURE PRESSURE EQUAL TO AMBIENT STATIC PRESSURE; TURBINE ADIABATIC EFFICIENCY, 80%; COMPRESSOR ADIABATIC EFFICIENCY, 70%; DUCT EFFICIENCY TAKEN FROM FIG. 2.

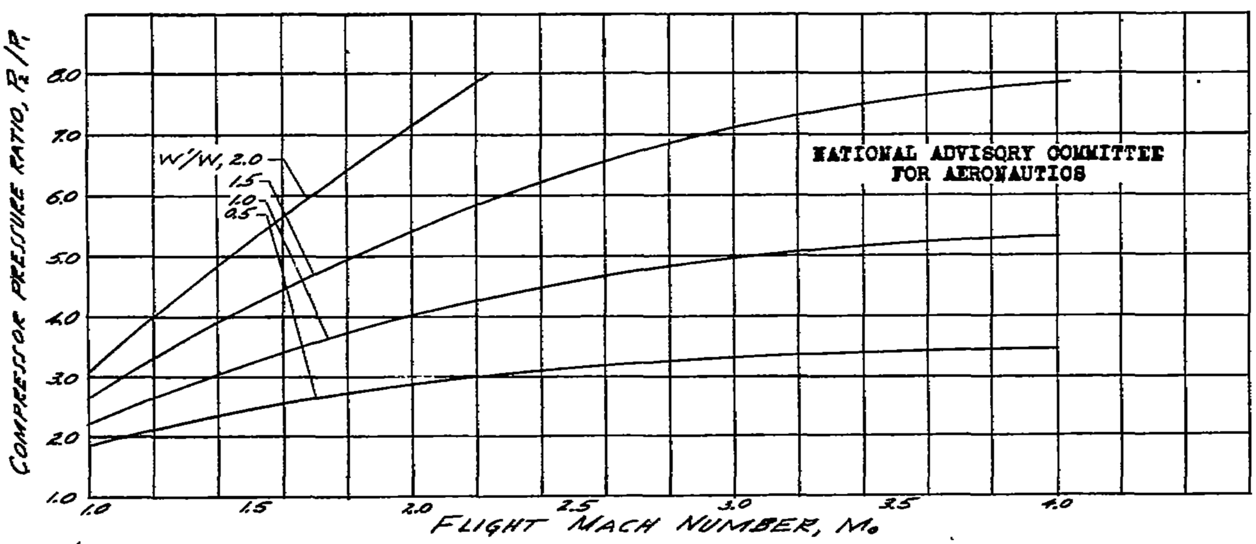


FIGURE 11.- COMPRESSOR PRESSURE RATIO AS A FUNCTION OF FLIGHT MACH NUMBER AND WEIGHT RATIO OF AUXILLIARY COOLING AIR TO VENTILATING AIR FOR SYSTEM V. ENCLOSURE PRESSURE EQUAL TO AMBIENT STATIC PRESSURE; COMPRESSOR ADIABATIC EFFICIENCY, 70%; TURBINE ADIABATIC EFFICIENCY 80%; INTERCOOLER EFFECTIVENESS, 90%; DUCT EFFICIENCY TAKEN FROM FIGURE 2.

[redacted]

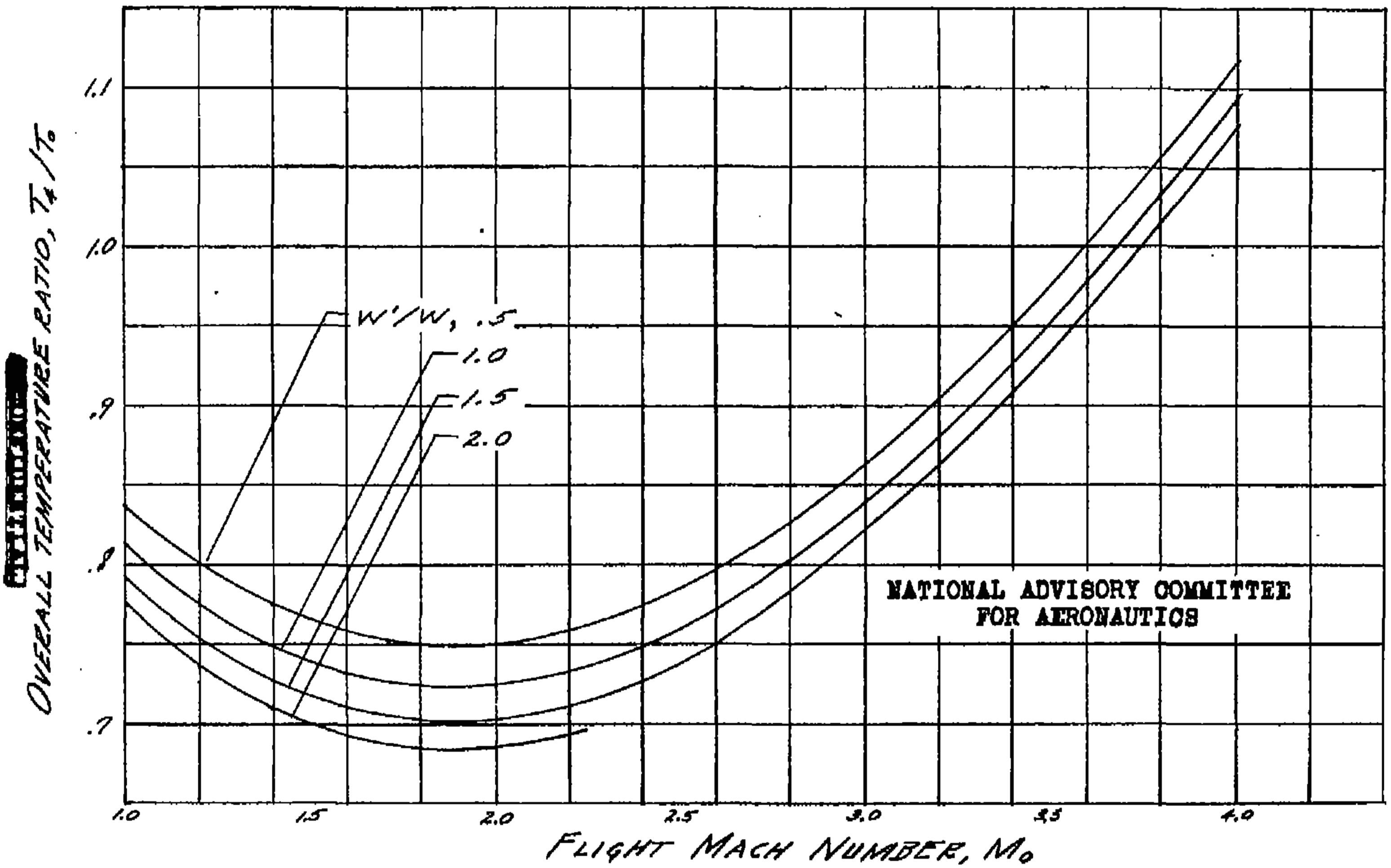


FIGURE 12.– OVERALL TEMPERATURE RATIO OF VENTILATING AIR FOR SYSTEM V AS A FUNCTION OF FLIGHT MACH NUMBER AND WEIGHT RATIO OF AUXILLIARY COOLING AIR TO VENTILATING AIR. ENCLOSURE PRESSURE EQUAL TO AMBIENT STATIC PRESSURE; COMPRESSOR ADIABATIC EFFICIENCY, 70%; TURBINE ADIABATIC EFFICIENCY, 80%; INTERCOOLER EFFECTIVENESS, 90%; DUCT EFFICIENCY TAKEN FROM FIG. 2.

[redacted]